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Brans–Dicke Models with Time-Dependent Cosmological Term

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More general solutions than those presented by Bertolami are deduced in the Brans–Dicke cosmology, endowed with a time-dependent cosmological term, for a Robertson–Walker metric and a perfect fluid obeying the perfect gas law of state.

Bertolami (1986) introduced in Brans–Dicke cosmology a time-dependent cosmological term, having in mind that it should explain why the present value of Λ is 10^{50} times smaller than in the Glashow–Salam–Weinberg model (Abers and Lee, 1973) and 10^{107} times smaller than in a GUT theory (Langacker, 1981). He solved it for the Robertson–Walker metric, in the $p=0$ and $p=\rho/3$ cases, where p is pressure and ρ is density. He found the following solution:

$$\Lambda = Et^{-2} \tag{1}$$

$$R(t) = At \tag{2}$$

where R is the scale factor, t stands for time, and E and A are constants. For $p=0$, he also found

$$\phi(t) = St^{-1} \tag{3}$$

where ϕ stands for the scalar field, S is a constant, and the gravitational constant G is given by

$$G = a\phi^{-1}$$

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where

$$a = \frac{4+2w}{3+2w} > 0$$

w is the Brans–Dicke coupling constant, and the positive sign of a is necessary in order that a given mass bend light in the correct direction.

For $p = \rho/3$, Bertolami found

$$\phi(t) = \mathbb{C}' t^{-2}, \qquad \mathbb{C}' = \text{const} \tag{4}$$

In what follows, we shall obtain more general results, solving Bertolami's equations for a perfect gas law of the type

$$p = \alpha\rho \tag{5}$$

where α is a constant. Our solutions satisfy relation (1) for Λ, which we take for granted at the beginning. We shall try, for tentative solutions, the constant-deceleration parameter laws studied by Berman (1983) and Berman and Gomide (1988), whose formulas are

$$H = \frac{\dot{R}}{R} = \frac{1}{mt} \tag{6}$$

$$H = DR^{-m} \tag{7}$$

$$R(t) = (mDt)^{1/m} \tag{8}$$

where H is Hubble's parameter; D and m are nonnull constants, and we have

$$m = q + 1 \tag{9}$$

where $q = -\ddot{R}R/\dot{R}^2 =$ deceleration parameter. Here, dots stand for one time derivative each. Bertolami's (1986) equations are

$$\frac{\ddot{\phi}}{\phi} + 3\,\frac{\dot{R}}{R}\,\frac{\dot{\phi}}{\phi} + \frac{2\Lambda}{3+2w} - \frac{2\phi}{3+2w}\cdot\frac{\partial\Lambda}{\partial\phi} = \frac{8\pi a}{\rho(3+2w)}(\rho - 3p) \tag{10}$$

$$\frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = \frac{8\pi a\rho}{3\phi} - \frac{\dot{\phi}\dot{R}}{\phi R} + \frac{w\dot{\phi}^2}{6\phi^2} + \Lambda \tag{11}$$

$$\dot{\rho} = -3\,\frac{\dot{R}}{R}(\rho + p) \tag{12}$$

$$\begin{aligned} &\frac{-8\pi a}{\phi^2}\,\dot{\phi}\rho + \left(w\frac{\dot{\phi}}{\phi} - 3\,\frac{\dot{R}}{R}\right)\frac{\ddot{\phi}\phi - \dot{\phi}^2}{\phi^2} - \frac{3}{R^2}(\ddot{R}R - \dot{R}^2)\,\frac{\dot{\phi}}{\phi} \\ &= \frac{\partial\Lambda}{\partial\phi}\,\dot{\phi} - 3\,\frac{\dot{R}}{R}\left(\frac{8\pi a\rho}{\phi} + \frac{w\dot{\phi}^2}{2\phi^2} - 3\,\frac{\dot{R}\dot{\phi}}{R\phi} - \Lambda\right) \end{aligned} \tag{13}$$

We shall try solutions for ϕ of the type

$$\phi = St^{A} \tag{14}$$

where A and S are constants. From (12) and (5), we obtain

$$\rho = \mathbb{C}t^{-(3/m)(1+\alpha)}, \qquad \mathbb{C} \text{ a constant}$$

Imposing

$$A = 2 - \frac{3(1+\alpha)}{m} \tag{15}$$

we obtain, from (10), (11), and (13), the following conditions for $k = (0, \pm 1)$:

$$1 + kD^{-2} = \frac{8\pi a\mathbb{C}}{3S} - A + \frac{w}{6}A^2 + E \tag{16}$$

$$A(A-1) + \frac{3}{m}A + \frac{2E}{3+2w} + \frac{4E}{A(3+2w)} = \frac{8\pi a(1-3\alpha)\mathbb{C}}{(3+2w)S} \tag{17}$$

$$\frac{8\pi a\mathbb{C}}{S}(3-A) + wA^2\left(\frac{3}{2m} - 1\right) + \frac{6A}{m} - \frac{9A}{m^2} = E\left(\frac{3}{m} - 2\right) \tag{18}$$

If $k = \pm 1$, we must also impose $m = 1$, which means that relation (2) is obeyed in a more general context than Bertolami's.

If $k = 0$, we need not particularize m, and we can find a solution were E, $\mathbb{C}/S$, w, and A are determinable in terms of α and m, which remain arbitrary.

As we can observe, Bertolami's solution for Λ is valid in the more general approach given here. Berman *et al.* (1989) found the same solution (1) in the static case, and Berman (1990) also retrieved it in another model for the static case. We conclude that the relation

$$\Lambda \propto t^{-2}$$

plays an important role in cosmology.

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REFERENCES

Abers, E. S., and Lee, B. W. (1973). *Physics Reports*, **9**(1), 1.
Berman, M. S. (1983). *Nuovo Cimento*, **74B**, 182.

Berman, M. S. (1990). *International Journal of Theoretical Physics*, **29**, 567.
Berman, M. S., and Gomide, F. M. (1988). *General Relativity and Gravitation*, **20**, 191.
Berman, M. S., Som, M. M., and Gomide, F. M. (1989). *General Relativity and Gravitation*, **21**, 287.
Bertolami, O. (1986). *Nuovo Cimento*, **93B**(1), 36.